EPISTEMOLOGY AND MATHEMATICAL REASONING

BY JAMES D. NICKEL

All knowledge is founded upon the fear of the Lord (Proverbs 1:7; 9:10). It is only in the light of God's Word that we can understand or know anything rightly (Psalm 36:9). Hence, *man knows by revelation*. The study of the theory of knowledge (how one comes to know and how one justifies that fact that he does know) is called *epistemology*, a philosophical term.

The discipline of mathematics is a unique interplay between inductive and deductive reasoning. The purpose of this essay is to explore the rudiments of the nature of these two types of reasoning.

Inductive reasoning is discovery by the "look and see" approach. It is, in effect, the "search for patterns." The Latin phrase for induction is *a posteriori* meaning "dependent upon observation, experience, or sense perception"). By this form of reasoning, you observe the "particulars" of a given situation and discern a general pattern or law (*empiricism*). Induction characterizes the scientific method, a set of techniques used for solving problems involving understanding the ways (i.e., patterns or laws; the *unity* in *diversity*) in which God's covenantal faithfulness is displayed in the ordinances of the physical creation (read Jeremiah 31:35). The "scientific method" involves several steps:

> Induction: "Look and see."

> Induction is reasoning from the particular to the general.

1. The construction of preliminary hypotheses or an attempted explanation defining a pattern observed.
2. The formulation of explanatory hypotheses or a formal explanation of the pattern observed.
3. The deduction of consequences from hypotheses or answering the question, "What can be predicted from these patterns?"
4. The testing of the consequences deduced or testing those predictions for accuracy.
5. The application of the theory thus confirmed to further problems or applying the pattern or law to new situations and thereby showing how this *one* law can describe and predict *many* situations.

Most people (including scientists) accept these steps without question. It is the way "things are done" in the world of science. In mathematics, very few mathematicians (including math professors) ever ask the question, "What justifies the use of theorems?"

The Biblical Christian seeks justification.

- How can we justify scientific law?
- On what basis can we define patterns and then use those patterns to predict?

> The essence of doing mathematics is the "search for pattern," either in number relationships or in scientific measurements.

These questions deal with the nature of knowledge or epistemology. Biblically, we answer these questions in terms of the doctrine of creation. The physical world, as the creation of God, will reflect His character (i.e., what is made reflects the one who made it). God reveals Himself to us in Scripture (known as verbal propositional truth) as a personal God, an infinite God, an eternal God, a holy God, a loving God, a good God, a true God, a Triune God. God, being triune, is the ultimate "One and the Many" (i.e., Unity in Diversity: *three* persons–Father, Son, and Holy Spirit, *one* essential nature). It is because of His triune nature that we can observe the "unity in diversity" of the physical creation. The created order is a proximate reflection of the ultimate "Unity in Diversity." Therefore, t*he revelation of the Biblical God justifies the possibility of scientific law* (i.e., law-like patterns in the physical world). In Genesis 2:18-20, God told Adam to observe the created order (e.g., the animal world) and then to name (or classify) that order (give it a name that reflects its characteristics). This naming process is the essence of the scientific method: *observe* and *classify*.

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The unbeliever, because of his ethical rebellion, refuses to submit to the one, true, and living God. Instead of seeing God as the Absolute (the *only* One worthy of worship), the unbeliever posits nature as absolute (read Romans 1). The unbeliever worships the creation rather than the Creator. Because of this ethical rebellion, the scientific method (in short, empiricism) has become in the modern world the *one and only pathway to true knowledge* (philosophically, this is called *positivism*). Anything that cannot be tested or proved in the laboratory (e.g., the existence of God) is therefore outside the bounds of true, verifiable knowledge. An incredible observation about many modern scientists is that they use the gifts God has given them (their minds and the law-like structures of the physical creation) to prove that God does not exist. As the Christian philosopher Cornelius Van Til (1895-1987) once said, and I am paraphrasing, "The unbeliever must first sit on God's lap if he wants to slap His face."

The inductive method of reasoning, by its very nature, has limitations. "Particulars" are the basis for pattern recognition but you cannot investigate the universality of the particulars. In other words, man is not omniscient and he cannot investigate all possibilities. There might be, somewhere, a counterexample to his pattern! In other words, all conclusions based upon inductive reasoning can be "disproved" by *one* counterexample.

For example, iron, copper, brass, oil, and other substances expand when heated. Hence, one could conclude that all substances expand when heated. There is a counterexample. Water, when heated from 0° to 4°C, does not expand; it contracts.

The following Presidents of the United States all died in office (natural death or assassinated). Date of election.

William Henry Harrison	1840
Abraham Lincoln	1860
James Garfield	1880
William McKinley	1900
Warren G. Harding	1920
Franklin D. Roosevelt	1940
John F. Kennedy	1960

Is it reasonable to conclude that the President elected in 1980 will die in office? No, President Ronald Reagan (elected 1980) did *not* die in office.

Deduction: "Stop and think."

In contrast to the "look and see" of induction, deductive reasoning is discovery by the "stop and think" approach. The Latin phrase for deduction is *a priori* meaning "independent of observation, experience, or sense perception." Deduction starts with one or several *premises* (in mathematics, these premises are called *axioms* or *postulates*) and then uses the method of logical reasoning to infer a *conclusion* from these premises. For example, Sir Arthur Conan Doyle's (1859-1930) fictional sleuth Sherlock Holmes was noted for his uncanny ability to use deductive reasoning to solve criminal cases. We need to place a "buyer beware" sign on deduction. Fallen man can easily absolutize deductive reasoning; i.e., the worship of the capabilities of the mind instead of the Maker of the mind. When man does this, he embraces *rationalism*, the attitude that man's *autonomous* (self-law) reason is his *final authority*, in which case he can deny or ignore divine revelation.[1]

Deduction is reasoning from the general to the particular.

[1] The Biblical Christian position is that man is rational because man is created in the image of a rational God. Because man is rational, man can engage his mind in his thoughts about God, the creation, and himself. This thinking is *not* rationalistic (adjudicat-

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Deductive reasoning is the "bread and butter" of mathematics. By it, man can construct a wonderful and interconnected system, but this system is never absolute in terms of the one and only pathway to all knowledge or truth. In 1931, the mathematician Kurt Gödel (1906-1978) showed that the deductive systems of mathematics have limitations; i.e., there are statements that are *true* in mathematics that *cannot* be proven using the methods of deduction![2]

I hope that you see that the study of mathematics has far-reaching ramifications in terms of the nature and justification of knowledge. For the Biblical Christian, induction and deduction are *tools* (gifts of God) whereby we can unearth the treasures of God's created order. As we discover these treasures, we are never to make them or the method of discovery absolute, for this is idolatry. We submit all of our findings to the light of the Creator God and His Word (for this is what "fearing the Lord" means). When we see them in terms of the absolute that is God, then we are truly free to enjoy and use them as faithful stewards under God.

Induction and deduction have borne wonderful fruits in the history of science. For example, the Christian astronomer Johannes Kepler (1571-1630) used induction to discover his famous three laws of planetary motion.[3] Later, Sir Isaac Newton (1642-1727), also a student of Scripture, used the basic laws of motion and gravitation as axioms, *and from these he deduced Kepler's three laws of planetary motion!*

A BRIEF LESSON IN GREEK

The ancient Greeks (ca. 600-300 BC) were the first to introduce *systematic* methods of deduction. Their goal was to use methods of reasoning to discern *order* in a universe. Contrary to the Biblical Christian worldview where the infinite, personal, wise, and Triune God is the source and sustainer of this order, the Greeks understood this order to be governed by inscrutable and ultimately inexplicable *fate*. To the Greeks, human reason was the *sole* means by which man's purpose, meaning, and existence could be justified. Reason was therefore their *salvation* from a world bounded by finiteness and death. Instead of seeing reason as a tool, they saw it as God (i.e., the ultimate standard). For the Biblical Christian, we can learn two lessons from the Greeks: (1) how *not* to use reason as the ultimate standard (i.e., *rationalism*) and (2) how to use the *tool* of reason *effectively under God.*

> Reasoning leads us from premises to conclusions; it cannot start without premises; ... we must believe that we have an inner sense of values which guides us as to what is to be heeded, otherwise we cannot start on our survey even of the physical world .... At the very beginning there is something which might be described as an act of faith–a belief that what our eyes have to show us is significant.
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> Arthur Stanley Eddington, *Science and the Unseen World: Swarthmore Lecture 1929* (New York: Macmillan Co., 1938), pp. 73-74.

Abstraction is grasping a common quality or qualities in different things (diversity) and forming a general idea (unity) there from. Abstraction, using a metaphor derived from the arithmetic of fractions, finds the "common denominator" in diverse things. Man can abstract in this manner *only because he is created in the image of the Triune God (the One and the Many) of Scripture.* For example, we note that the unifying or general principle of churches, houses, and skyscrapers is that they are all buildings. The unifying or general

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ing final authority to one's thoughts); it is thinking in dependence upon God or, as Augustine (354-430), Bishop of Hippo, once said, "Thinking God's thoughts after Him."

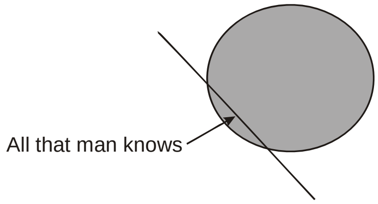
[2] Kurt Gödel, *On Formally Undecidable Propositions of Pincipia Mathematica and Related Systems* (New York: Dover Publications, [1931] 1992). For a "layman's explanation" of his proof, See Rózsa Péter, *Playing with Infinity: Mathematical Explorations and Excursions* (New York: Dover Publications, [1957, 1961] 1976), pp. 211-265.

[3] Kepler spent years studying astronomical data (the "particulars") to try to discover mathematical relations of "best fit."

principle of cows, cats, and dogs is that they are all animals. Look at the corner of your room. You can abstract the idea of a *straight line* from the *intersection* of two vertical walls. You can abstract the idea of a *point* from the *intersection* of the horizontal ceiling and two vertical walls. You can abstract the idea of a *plane* from the surface of a wall or a ceiling.

Abstraction is closely related to inductive thinking (i.e., reasoning from the particular to the general). The difference is that induction finds the "common denominator" *in the same class of things*. For example, from our observation of dogs, we can conclude that "all dogs bark" or that "all Doberman pinschers are dogs." As a method of science (i.e., empiricism), induction is observing repeated instances of the *same* phenomenon *concluding that this phenomenon will always occur*. You cannot prove, in an absolute sense, that a conclusion is *always* true based on a *limited* number of instances. When it comes to proving the existence of the Biblical God, the method of induction (reasoning from the particulars of experience) will fail precisely because man cannot test everything (i.e., all reality) in a scientific lab. God is real and He can be known, but only on His terms. Those terms are given to man, not in a scientific lab, but in *revelation* (i.e., the Bible).

Deductive proof, in review, is a logical argument that moves from a given or general *premise* (an assumption that *may or may not be true*) to a particular *conclusion* (also called a theorem, meaning "a subject to think about") in such a way that no flaw can be found in each step of the argument. *Deduction combines accepted facts in a way that compels acceptance of the conclusion*. The Greeks called these premises *axioms* (meaning to "think worthy") or postulates (meaning "a thing demanded"). The Greeks used the term *axiom* to reflect a *general* "worthy statement." For example, "the whole is greater than the part" is a statement worthy of general acceptance. They used the term *postulate* to reflect a specific "thing demanded" by the subject under study. If that subject was geometry (meaning "the measure of the earth"), then an example of a postulate would be "two points determine a straight line." The Greeks, by the way, fit most, if not all their mathematics, in terms of the rules and structure of *Euclidean* (after a Greek geometer named Euclid, who lived in the 3rd century BC) or *plane* (primarily two-dimensional) geometry.[4]



Man's limited knowledge; man can know *truly*, but never *exhaustively*.

Here is a simple example of deductive reasoning. Let our preimse be "all dogs bark." Our next premise is that "a Doberman pinscher is a dog." Therefore, my inescapable conclusion (i.e., theorem) is that "all Doberman pinschers bark." A *corollary* is a statement that follows from a theorem. For example, if my neighbor's dog is a Doberman pinscher, then my neighbor's Doberman pinscher barks.

Deduction can also be unreliable (the ancient Greeks, to their detriment, refused to recognize this). For example, let's start with some basic definitions of arithmetic. An integral number is a whole number (a complete entity); e.g., 1, 2, 3, 4, 5, etc. A *remainder* means "something left over." When you divide 12 by 5, you get 2 with a remainder of 2. A *multiple* of a number is a number that contains another number an integral number of times *without a remainder*; e.g., 12 is a multiple of 3 because it contains another number, i.e., 4, such that $3 \times 4 = 12$ (or 12 divided by 4 equals 3 without a remainder).

> I have never found a better expression than the expression 'religious' for this trust in the rational nature of reality and of its peculiar accessibility to the human mind. Where this trust is lacking science degenerates into an uninspired procedure. Let the devil care if the priests make capital out of this. There is no remedy for that.
> Albert Einstein, *Lettres à Maurice Solovine* (Paris, 1956), pp. 102-103.

[4] Included in Euclid's study is number theory (understood in the context of geometry) and solid, or three dimensional, geometry.

An *odd* number is any number that is *not* a multiple of two (e.g., 1, 3, 5, 7, 9, etc.). An *even* number is a multiple of 2 (e.g., 2, 4, 6, 8, etc.). Consider this premise: *The sum of two odd numbers is always even.* Let x and y stand for two odd numbers. We are doing algebra when we let general symbols (i.e., letters) stand for specific numbers. If the sum of x and y is even, then we can conclude that that x and y are odd numbers. But, let $x = 6$ and $y = 8$. What is the sum? 14. Yes, the sum of two odd numbers is always even *but the sum of two even numbers is also always even.*[5]

There are two reasons why deduction is limited. First, we must start with basic premises. Remember, a premise is an *assumed* true statement. Many of our assumptions will suffer from some limitations. For example, we do not know, for certain, that there is a cure for cancer. Therefore, we cannot state as a premise that there is a cure for cancer. Second, with induction you can find a pattern (or conclusion) more easily than using the logical efforts of deductive proof. For example, if you know Euclidean geometry, then you can deductively prove that the measure of the three angles of a triangle sum to 180°. This proof is not as simple as it may seem and you need to invoke an Euclidean postulate, the parallel postulate, that has a checkered and somewhat controversial history! If you do *not* know Euclidean geometry, then measure the angles of three different triangles and come to your conclusion that way (I suggest you try it as an exercise). The Italian scientist Galileo (1564-1642) used induction to find the area of complex figures. Using cardboard, he replicated a complex figure and compared its weight with a cardboard figure whose area he did know. Using the premise that relative weights are equivalent to relative areas, he could determine the areas of these complex figures.

The Greeks also introduced another valuable reasoning tool. It is called *reductio ad absurdum,* meaning "reduction to the absurd."[6] You can, by this method, prove the truth of a premise by: (1) assuming the *opposite* of the premise to be true and (2) demonstrating that this assumption eventually leads to a logical conundrum or contradiction. Here is what we want to prove: *Some dogs do not bark constantly.* We already know other things (premises) about dogs, like all dogs are animals, and all animals must eat and sleep. Let's see where our reasoning will take us. We will lay out our proof in two columns as follows:

STATEMENT	REASONS
1. All dogs are animals.	Premise
2. All animals must eat and sleep.	Premise
3. All dogs must eat and sleep.	Deduction from Step 1 and 2
4. Some dogs bark constantly.	Opposite of what we want to prove
5. Dogs that bark constantly cannot eat or sleep.	Premise (from induction)
6. Some dogs do not eat or sleep.	Deduction from Step 4 and 5

Step 6 finishes our proof and we can conclude that some dogs do not bark constantly. Why? Step 6 (some dogs do not eat or sleep) contradicts Step 3 (all dogs must eat and sleep). Hence, our assumption (Step 4) is false and the opposite of what we assumed in Step 4 (i.e., what we wanted to prove) is true. You may have to think through this paragraph several times to "get" this!

[5] We prove by deduction that the sum of two odd numbers is even by noting first, as a definition, that 2n (where n is an integral or whole number) represents an even number and 2n + 1 represents an odd number. Let 2j + 1 and 2k + 1 be two odd numbers where j and k are whole numbers. Adding them, we get $(2j + 1) + (2k + 1) = 2j + 2k + 2 = 2(j + k + 1)$. Since j and k are whole numbers, then j + k + 1 is a whole number. Hence, 2(j + k + 1) is of the form 2n where n is a whole number (n = j + k + 1) and we have therefore justified, via deduction, that the sum of two odd numbers is an even number. We use the same method to prove that the sum of two even numbers is always even.

[6] There are different schools of mathematical thought (regarding its foundations) and not all these schools agree that *reductio ad absurdum* is a valid method of proof!

Keep this method in mind for you will encounter it often in future math classes (especially Geometry). Mathematicians primarily employ the tool of deduction. Christian philosopher Cornelius Van Til (1895-1987) used *reductio ad absurdum* in his famous transcendental proof of the existence of God. What he does is that he starts his proof with the assumption that the God of Scripture *does not exist*. Then, he reasons to a contradiction (in fact, a whole host of contractions; e.g., how reality is falsified when the existence of God is denied). From these contractions, he concludes that his assumption is false and what he wanted to prove (i.e., the existence of God) is true.

By relying upon and exploiting deductive reasoning, mathematicians have obtained results that would be very difficult or even impossible to obtain by other methods. For example, the Greek mathematician Eratosthenes (ca. 276–ca. 195 BC) used a few measurements plus deductive reasoning to calculate the *circumference of the earth*. According to mathematics historian Morris Kline (1908-1992):

> "… man's reason can encompass distances, sizes, sounds, and temperatures beyond the range of the senses. More than that, reason can contemplate phenomena which transcend the senses and even the imagination. Mathematics has thereby been able to create spaces of arbitrary dimension and to predict the existence of imperceptible radio waves. And because mathematics has confined itself to the soundest methods of reasoning man has, the results of mathematics have endured, whereas even some of the most magnificent theories of science have had to be discarded."[7]

Because deductive reasoning relies upon reliable starting points, it is important to establish their verity. Because deductive and inductive reasoning are gifts from God, the ultimate "starting point" for thinking is God's revelatory gifts (Proverbs 1:7; 9:10; Psalm 36:9). It is only in this *fear of the LORD* that the Biblical Christian can *truly* value and justify these methods of thinking.

[7] Morris Kline, *Mathematics and the Physical World* (New York: Dover Publications, [1959] 1980), p. 19.