What is a Googol? … And Other Large/Small Numbers

by James D. Nickel

With the rising US national debt and massive federal "stimulus" packages, large numbers are on the mind of many. Some names of large numbers, as powers of ten, are:

- 10^6 = million
- 10^9 = billion (bi: 2)[1]
- 10^{12} = trillion (tri: 3)
- 10^{15} = quadrillion (quarter: 4)
- 10^{18} = quintillion (quintus: 5)
- 10^{21} = sextillion (sex: 6)
- 10^{24} = septillion (septem: 7)
- 10^{27} = octillion (octo: 8)
- 10^{30} = nonillion (novem: 9)
- 10^{33} = decillion (decem: 10)

How much is $1 million? Suppose someone offered you $1 million in $1 bills if you could carry it away. Will you be able to do so? You can fit five $1 bills on one piece of paper (8½ by 11 inches). There are 500 sheets of paper in one ream and two reams of paper weigh 9 pounds. $\frac{1,000,000}{5} = 200,000$ sheets and $\frac{200,000}{500} = 400$ reams. If 2 reams weigh 9 pounds, then we have a proportion:

$$\frac{2}{9} = \frac{400}{x} \Leftrightarrow 2x = 3600$$
$$x = 1800$$

The weight of the $1 million dollars would be nearly 1 ton (2000 pounds)! That's a little outside of the range of a champion weightlifter!

Federal deficits and spending are "only" in the billions and trillions range, amounts very far beyond our grasp. To get an idea, first consider a packet of *one hundred* $100 bills. It is less than 0.5 inches thick and contains $10,000. $1 million dollars (100 packets of $10,000) would fit in a grocery bag. $100 million fits neatly on a standard pallet. $1 billion dollars fits on ten pallets, but to understand $1 trillion dollars, a million million, you need to *double-stack* these pallets, 50 columns by 100 rows!

> One of the problems in trying to comprehend federal spending is that the units involved–billions of dollars–are so large as to be almost meaningless to many citizens. To visualize what a billion dollars means, imagine that some organization had been spending a thousand dollars a day *every day since the birth of Christ.* They would not yet have spent a billion dollars. In the year 2000 they would still be more than 250 million dollars short of one billion dollars. Government agencies of course spend not only one but many billions of dollars annually. To get a figure comparable to what the entire federal government spends annually, change the one thousand dollars per day to *half a million dollars a day*, every day since the birth of Christ. At the end of two thousand years the grand total would amount to less than three quarters of what the federal government spent in 1978 alone [In the early 21st century, federal spending is no longer projected billions, but in trillions or a *thousand* billions of dollars–JN].
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> Thomas Sowell, *Knowledge and Decisions,* p. 306.

[1] Billions and trillions are understood differently in Europe. The European billion is a million millions or 10^{12} and a trillion is 10^{18}.

WHAT IS A GOOGOL? … AND OTHER LARGE/SMALL NUMBERS

BY JAMES D. NICKEL

One of the largest named numbers is the googol = 10^{100} (1 followed by one hundred zeroes). Milton Sirotta, nephew of mathematician Edward Kasner, first coined this word and it was first mentioned in Kasner's book *Mathematics and the Imagination* (New York: Dover Publications, [1940, 1967] 2001). Extending his nephew's idea, Kasner coined the word *googolplex* to define an especially large number. 1 *googolplex* = $10^{10^{100}}$ or 10^{googol} (1 followed by 10^{100} zeroes!).

Googol has its own unique lore. It was the answer to the million-pound question on the British version of *Who Wants to Be a Millionaire* on 10 September 2001. A googol is greater than the number of particles in the known universe, which has been variously estimated from 10^{72} up to 10^{87}. The *Shannon number* is a rough estimate of the number of possible chess games, and it is more than a googol: 10^{120}. The Internet search engine Google was named after this number; the original founders were going for "Googol," but ended up with "Google" due to a spelling mistake.

How large is a *Googolplex?* To get an idea, let's first review the simple laws of exponents. First, we consider the *product law of exponents:* $b^x \times b^y = b^{x+y}$. Note that the base *b* must be the same in both factors. We see how this law develops by observing $2^3 \times 2^5 = (2 \times 2 \times 2)(2 \times 2 \times 2 \times 2 \times 2) = 2^8$.

$$2^3 = 8,\ 2^5 = 32 \text{ and } 2^8 = 256.$$

Hence, 8 ×3 2=256.

Second, we consider the *quotient law of exponents:* $\frac{b^x}{b^y} = b^{x-y}$ Again, the base *b* must be the same in both dividend and divisor. We see how this law develops by observing $\frac{2^5}{2^2} = \frac{2\times2\times2\times\cancel{2\times2}}{\cancel{2\times2}} = 2^{5-2} = 2^3$.

$$2^5 = 32,\ 2^2 = 4, \text{ and } 2^3 = 8.$$

$$\text{Hence, } \frac{32}{4} = 8.$$

A number is written in *Scientific Notation* if it is written in the form $a \times 10^b$ where $1 \le a < 10$. *a* is called the coefficient and *b* is called the exponent and ≤ means "less than or equal to." Here are some numbers written in Scientific Notation:

- $450 = 4.5 \times 10^2$
- $4500 = 4.5 \times 10^3$
- $4{,}500{,}000 = 4.5 \times 10^6$
- NB. $10^5 = 1 \times 10^5$

A googol is considerably less than the number calcuated in the ancient Greek story by Archimedes (ca. 287-212 BC) entitled *The Sand Reckoner*. The number he described was $10^{8\times10^{16}}$. A "little" googol is $2^{100} \approx 1.267\times 10^{30}$ and a little googolplex is $2^{2^{100}} \approx 10^{3.8\times10^{29}}$.

Getting "smaller," the radius of the Earth is a mere 6.37 million meters or 6.37×10^6 meters. An astronomical unit is the mean (average) distance from the Earth to the Sun. It is 149,569,800,000 meters or about 1.5×10^{11} meters. The diameter of our Milky Way galaxy is 10^{20} meters (100 quintillion) or 1×10^{20} meters.

The estimated number of stars in the universe is 10^{26}. There are about 10^{11} galaxies and there are about 10^{11} stars in each galaxy. We note that $10^{11} \times 10^{11} = 10^{22}$. Albert Einstein (1879-1955) increased that number by a factor of 10,000 or 10^4. Hence, $10^{22} \times 10^4 = 10^{26}$. We are not taking into consideration the number of radio stars, the stars we cannot see (about 10^{11} per galaxy)! With this is mind, reconsider Psalm 147:5 …

> "He counts the number of the stars; He calls them all by name, Great is our Lord, and mighty in power; His understanding is infinite."

What Wonder!

We can also write small numbers in Scientific Notation. The diameter of a poliomyelitis virus is 28 thousandths of a millionth of a meter or 0.000000028 or 2.8×10^{-8} meters. The diameter of an atomic nucleus is about 75 thousand-billionths of a meter or 0.000000000000075 meters or 7.5×10^{-14} meters. 1 nm (nanometer) = 1.0×10^{-9} meters. 1 Å (angstrom) = 1.0×10^{-10} meters. As you can see, Scientific Notation is a shorthand way of writing very large and very small numbers.

With this background behind us, let's engage the question, "How large is a Googolplex?" Note carefully that a googolplex is 10^{googol} or 1 followed by a "googol" of zeroes. "Imagine" printing the digits of a googolplex in unreadable, 1-point font (the text in this essay is written in a 12-point font). A letter written in 1-point font would measure 0.3514598 mm. Using Scientific Notation, it would take 3.5×10^{96} meters to write a googolplex in one point font. The known universe is estimated at 7.4×10^{26} meters in diameter. The distance to write the digits of a googolplex in 1-point font would be about 4.7×10^{69} times the diameter of the known universe. I'll leave it up to you to *do the math* to verify this. Talk about a large number!

Let's use the mathematics of exponents and Scientific Notation to consider the answer to one final question, "What would happen if you folded a piece of paper 50 times?" This time I will *do the math.*

We first note that 500 sheets (one ream) of paper measure 2 inches in height. In folding, we have a binary sequence:

- 1 fold = 2 sheets = 2^1
- 2 folds = 4 sheets = 2^2
- 3 folds = 8 sheets = 2^3
- 50 folds = 2^{50} sheets ≈ 1.126×10^{15} sheets.

Of course, folding paper this many times is impossible. We continue our mathematics as a "mind excursion" only. Since 500 sheets equal one ream of paper, we have:

$$\frac{1.126\times10^{15}}{500} = \frac{11.26\times10^{14}}{5\times10^2} =$$

$$2.252\times\frac{10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times \cancel{10\times10}}{\cancel{10\times10}} =$$

$$2.252\times10^{14-2} = 2.252\times10^{12} \text{ reams of paper.}$$

Since one ream measures 2 inches in height, then the height of 50 folds is $\left(2.252\times10^{12}\right)(2)$ = 4.504×10^{12} inches. Since there are 12 inches in one foot, then $\dfrac{4.504\times10^{12}}{12} = \dfrac{4.504\times10^{12}}{1.2\times10^1} =$

$3.753\times10^{12-1} = 3.753\times10^{11}$ feet. Since there are 5280 feet in one mile, then: $\frac{3.753\times10^{11}}{5280} = \frac{37.53\times10^{10}}{5.28\times10^{3}} = 7.11\times10^{7} = 71,100,000$ miles.

Gulp! The distance from the Earth to the Sun is about 93,000,000 miles!