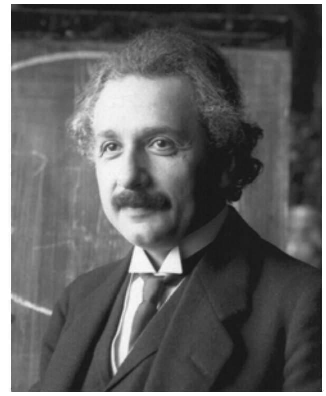
THE NATURE OF MATHEMATICS

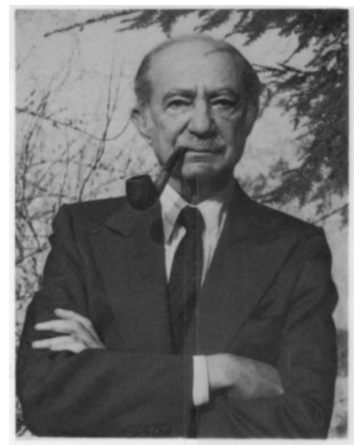
BY JAMES D. NICKEL

God is the Creator of the physical world, a *concrete* and objective world that stands outside of man and a world that reflects mathematical patterns, and the human mind, a mind that can think mathematics *in the abstract.* This one sentence explains why mathematics *works,* a truism that is a *mystery* to a host of philosophers, scientists, and mathematics.[1] One of the greatest scientists of the 20th century, Albert Einstein (1879-1955), reflected on this conundrum, "The eternal mystery of the world is its comprehensibility."[2] Mathematics historian Morris Kline (1908-1992) amplifies Einstein's succinct remarks:



Albert Einstein (Public Domain)

> But where and what are the physical agents that produce the effects mathematics describes? … There are no answers … Why does mathematics work? We are faced with a mystery … The study of mathematics and its contributions to the sciences exposes a deep question. Mathematics is man-made. Yet with this product of his fallible mind man has surveyed spaces too vast for his imagination to encompass, he has predicted and shown how to control radio waves which none of our senses can perceive, and he has discovered particles too small to be seen with the most powerful microscope. Cold symbols and formulas completely at the disposition of man have enabled him to secure a portentous grip on the universe. Some explanation of this marvelous power is called for."[3]



Morris Kline,
Courtesy of Oxford University Press

One historical example of this relationship between mathematics and the physical world is Joseph Black's (1728-1799) discovery of carbon dioxide gas, a discovery made by solving simultaneous equations.

Black, a Scottish chemist, was a pioneer in quantitative chemistry (applying measurement and mathematics to chemical reactions[4]). He was one of the first scientists (or natural philosopher as they were known in the 18th century) to *weigh* these substances (called reagents) *before* and *after* a chemical reaction.

In his 30s, Black investigated the uses of magnesia (also known as "milk of magnesia") as an antacid and laxative. He noticed that the *exact same weight loss* occurred in two different experiments involving *Magnesia Alba* (known today as calcium carbonate).

In one experiment, he added acid the Magnesia Alba, and in the other, he heated it. Here is what he did:

Experiment 1 (add acid): Magnesia Alba + acid →[5] residue + weight loss
Experiment 2 (add heat): Magnesia Alba + heat → residue + weight loss

We are now going to rewrite the description of the two experiments as mathematical equations. We first set the variables:

---

[1] This truism did *not* confound the leaders of the Scientific Revolution; e.g., Galileo, Kepler, and Newton. That God was the created of the physical world and the mind of man was a given that undergirded all their creative works.
[2] Albert Einstein, *Out of My Later Years* (New York: Citadel Press, [1950, 1956, 1984] 1991), p. 61.
[3] Morris Kline, *Mathematics and the Physical World* (New York: Dover Publications [1959] 1981), p. ix.
[4] Chemistry is the study of the reaction of one substance with another.
[5] → is a symbol that stands for "what happens on the left produces the reactants on the right." This symbol is similar to the equal sign in mathematics).

M = Magnesia Alba
a = acid
h = heat
r_1 = residue from Experiment 1
r_2 = residue from Experiment 2
w = weight loss

We replace → with the equal sign (=) and write two equations:

Equation 1: $M + a = r_1 + w$
Equation 2: $M + h = r_2 + w$

Black looked at these two chemical "equations" and decided to apply algebra to them. Since he was dealing with weights, he knew that heat has zero weight. He let h = 0 and rewrote the two equations (h drops out since M + h = M + 0 = M):

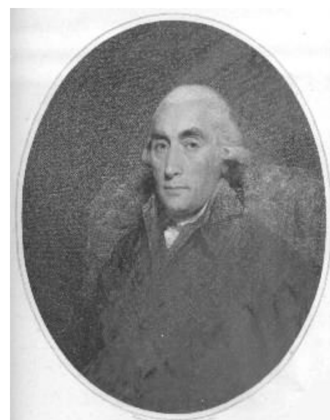
Equation 1: $M + a = r_1 + w$
Equation 2: $M = r_2 + w$

Next, Black subtracted Equation 2 from Equation 1. He got an equation that gives the solution to the variable *a* (the acid).

$a = r_1 – r_2$

Black then noticed, on the right side of the equation, one term was subtracted from the other (i.e., $r_1 – r_2$). He added r_2 to both members of the equation.

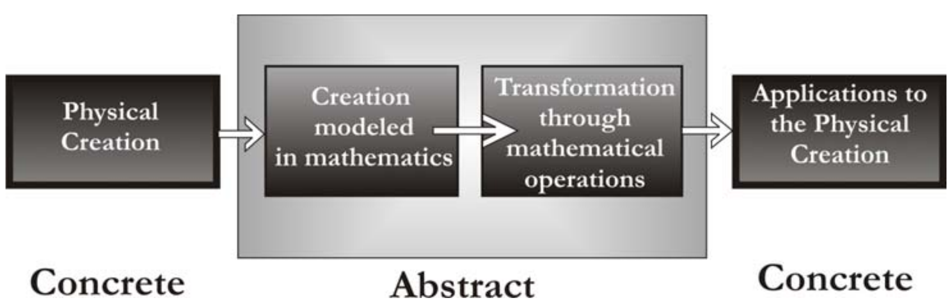
$a + r_2 = r_1$


Joseph Black, Public Domain

Black now tested his result empirically. He added the residue (r_2) to the acid and, lo and behold, he produced residue (r_1)! The algebra he applied to chemical "equations" *connected* to the world of his experiments. He showed that the weight loss was really a new kind of gas. He called this gas "fixed air," a gas we know today as *carbon dioxide* (CO_2).

In Black's case, a physical idea (concrete) suggested a mathematical solution (abstract). Once he found this solution, it returned to the "real world" of his experiment (concrete) as an exact "fit."

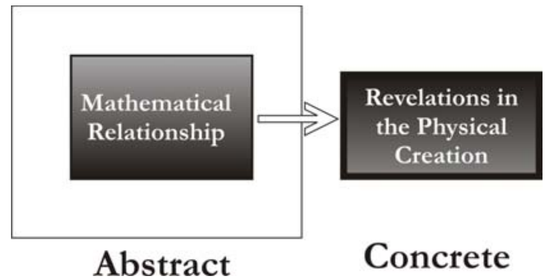
This method, from the concrete to the abstract and back to the concrete, is a common occurrence, not only in quantitative chemistry, but in mathematical physics. The concrete, the physical world, and the abstract, the mathematical mind of man, *cohere* (i.e., the unity in the diversity of between the concrete and the abstract) because of a common Creator!



THE NATURE OF MATHEMATICS

BY JAMES D. NICKEL

Man in his thinking can discern patterns in the physical creation (the concrete realm) because both realms (the physical world and the mind of man) are *imago dei* (in the image of God, their common Creator). Man models these patterns using mathematical symbols and he manipulates those symbols using mathematical operations and order (the abstract realm). By this analysis (modeling and manipulation), he can develop new mathematical relationships (still in the abstract realm). Then these new mathematical relationships find application in the physical world (back to the concrete realm).



Mark Levi, in his excellent book *The Mathematical Mechanic,* provides a multitude of real word situations, some very common, that show how a mathematical principle can be illustrated by a particular physical problem. In other words, instead of starting with a physical problem and abstracting it to a mathematical one, his book demonstrates the *reverse procedure!* He explicates the truism that looking for a physical meaning of a mathematical relationship (law or principle or theorem) is often amazingly revelatory!

A few of his examples include:

> Physical ideas can be real eye-openers and can suggest a strikingly simplified solution to a mathematical problem. The two subjects [mathematics and the physical world–JN] are so intimately intertwined that both suffer if separated … It may be argued that the separation of the two subjects is artificial.
>
> Mark Levi, The Mathematical Mechanic: Using Physical Reasoning to Solve Problems (Princeton University Press, 2009), p. 2.

- The Pythagorean Theorem is revealed in the law of conservation of energy.
- The inequality $\sqrt{ab} \le \frac{1}{2}(a+b)$ is revealed in the flipping of a switch in a simple electrical circuit.
- The Gauss-Bonnet formula[6] is revealed in the motion of a bike wheel.
- The Riemann integral formula of the Calculus and the Riemann mapping theorem are revealed by observing fluid motion.

This is a fun book to explore! It confirms again, albeit from a relatively non-traditional viewpoint, the amazing connections (connections that Levi reports *but does not justify*) between abstract mathematical propositions generated in the mind of man and the physical world of God's making.

> Why does mathematics work in the material universe? … Why does mathematics work so magnificently as a model to explain our universe? Scientists use mathematical models of the physical world to make certain claims and predictions about the world. Why should this relationship between model and physical reality exist unless there is some underlying connection? If numbers are only objects of thought, then why are they so wonderfully useful in analyzing the material universe?
>
> Calvin C. Clawson, *Mathematical Mysteries: The Beauty and Magic of Numbers* (Cambridge, MA: Perseus Books, 1996), p. 52.

---

[6] In differential geometry, this formula unifies (or connects) the geometry of an object (in the sense of curvature) to its topology (in the sense of the Euler characteristic).